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Numerical modeling of heat and mass transfer during forced convection drying of rectangular moist objects

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Abstract

A two-dimensional analysis of heat and mass transfer during drying of a rectangular moist object is performed using an implicit finite difference method, with the convective boundary conditions at all surfaces of the moist object. The variable convective heat and mass transfer coefficients are considered during the drying process. The external flow and temperature fields are first numerically predicted through the Fluent CFD package. From these distributions, the local distributions of the convective heat transfer coefficients are determined, which are then used to predict local distributions of the convective mass transfer coefficients through the analogy between the thermal and concentration boundary layers. Also, the temperature and moisture distributions for different periods of time are obtained using the code developed to determine heat and mass transfer inside the moist material. Furthermore, the influence of the aspect ratio on the heat and mass transfer is studied. It is found that the convective heat transfer coefficient varies from 4.33 to 96.16 W/m^2 K, while the convective mass transfer coefficient ranges between 9.28×10^{-7} and 1.94×10^{-5} m/s at various aspect ratios. The results obtained from the present analysis are compared with the experimental data taken from the literature, and a good agreement is observed.

1. Introduction

Drying is a fundamental problem involving simultaneous heat and mass transfer under transient conditions resulting in a system of coupled nonlinear partial differential equations. Understanding the heat and mass transfer in the product will help to improve drying process parameters and hence the quality. A number of internal and external parameters influence drying behavior. External parameters include temperature, velocity and relative humidity of the drying medium (air), while internal parameters include density, permeability, porosity, sorption–desorption characteristics and thermophysical properties of the material being dried.

Mathematical modeling has now become a common practice in analyzing the drying phenomena due to the cost and time involved in experimental studies. Such a model is established with the governing differential equations coupled with the initial and boundary conditions. Since the governing equations are generally in nonlinear structure, their analytical solution is mostly unnecessary and hence, numerical solutions are sought. In the open literature, numerical studies of drying mechanisms are generally based on one- and two-dimensional models.

Balaban and Pigott [1] used a variable grid central finite difference method for solving simultaneous heat and moisture transfer equation with variable transport parameters to predict the time-dependent local moisture content and temperature variations within an infinite slab during drying. Dutta et al. [2] numerically studied drying characteristics of a spherical grain using the Crank–Nicholson implicit numerical procedure. Wang and Brenman [3] proposed a mathematical model of simultaneous heat and mass transfer for the prediction of moisture and temperature distributions during drying in a slab-shaped solid using a

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Nomenclature

AR	aspect ratio
C_p	constant pressure specific heat (J/kg K)
D	moisture diffusivity (m^2/s)
D_0	pre-exponential factor of Arrhenius equation (m^2/s)
h	heat transfer coefficient (W/m^2 K)
h_m	moisture transfer coefficient (m/s)
H	height (m)
k	thermal conductivity (W/m K)
L	length (m)
Le	Lewis number α/D
M	moisture content (kg/kg, db)
n	normal to surface
P	pressure (Pa)
s	surface coordinate
t	time (s)
T	temperature (K)
u, v	velocities in x and y direction (m/s)
x, y	coordinates

Greek symbols

α	thermal diffusivity (m^2/s)
ρ	density (kg/m^3)
μ	dynamic viscosity [Pa s]
θ	dimensionless temperature, $\frac{T-T_i}{T_d-T_i}$
ϕ	dimensionless moisture, $\frac{M-M_e}{M_i-M_e}$

Subscripts

cp	center point
d	drying air
i	initial
∞	free stream

finite-difference method. Murugesan et al. [4] carried out a one-dimensional finite element analysis to study temperature and moisture variation of porous materials during convective drying. Dietl et al. [5] developed a physical–mathematical and a numerical model describing the drying of a single solid based on the conservation of heat and enthalpy flow rates as well as mass flow rates for a differential control volume. Azzouz et al. [6] used two models of diffusion to evaluate the effective diffusivity. Hernandez et al. [7] proposed an analytical solution of a mass transfer equation with concentration-dependent shrinkage and constant average water diffusivity. Dincer et al. [8] developed a new moisture transfer correlation (Bi–Re) to determine moisture transfer parameters. Zili and Nasrallah [9] presented a numerical simulation scheme for forced convection drying of granular products using the finite volume method. Suresh et al. [10] numerically simulated incompressible, two-dimensional fluid flow, heat and mass transfer over a rectangular brick due to transient laminar mixed convection using the finite element procedure. Chua et al. [11] developed a numerical method solving the coupled heat and mass equations for liquid water vapor movements through a porous food material using the finite volume method. Hussain and Dincer [12] presented a 2-D numerical analysis of heat and moisture transfer during drying of a cylindrical object using an explicit finite difference approach.

The majority of the above listed studies assumed constant convective heat and mass transfer coefficients in the analysis and this may not reflect the reality. This is in fact the motivation for this original work. This study consists of the following essential parts: (i) analysis of the external flow and temperature fields by a CFD package, (ii) determination of the spatial variations of the convective heat transfer coefficients, (iii) calculation of the spatial variations of the convective mass transfer coefficients using the analogy between the thermal and concentration boundary layers, (iv) computation of the temperature and moisture distributions inside the moist material using a finite-difference based implicit numerical method, (v) repetition of the above studies by changing the aspect ratio, and (vi) validation of the present model with the experimental drying data taken from Velic et al. [13].

2. Modeling

2.1. Modeling of external flow and temperature fields of the drying fluid

Fig. 1 shows the problem domain, with the corresponding boundary conditions, for the evaluation of external flow and temperature fields of the drying fluid around the object subject to drying. The partial differential equations governing the forced convection motion of a drying fluid in a 2-D geometry are the mass, momentum and energy conservation equations. In the simplified case, thermal and physical properties are assumed to be constant (i.e., the variation of fluid properties with temperature has been neglected). Considering the flow incompressible, for a two-dimensional problem, the most general form of the Navier–Stokes equations is given as follows:

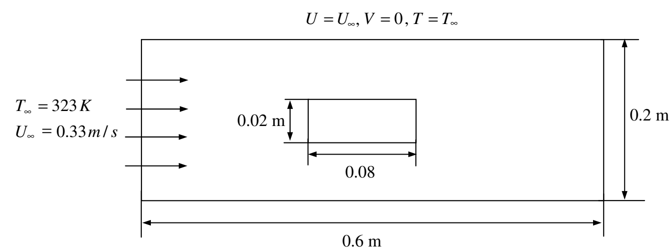

Fig. 1. The problem domain for the external field around the object.

The mass conservation (i.e., continuity) equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

The momentum equations are

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \tag{3}$$

The energy equation is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

Here, the Fluent V6.1.22 CFD package [15] based on the finite volume method is used to transform and solve these equations. The discretization scheme used is hybrid for the convective terms in the momentum and energy equations, and the SIMPLE algorithm for pressure–velocity coupling. The mesh is generated in the Gambit 2.1.6 preprocessor

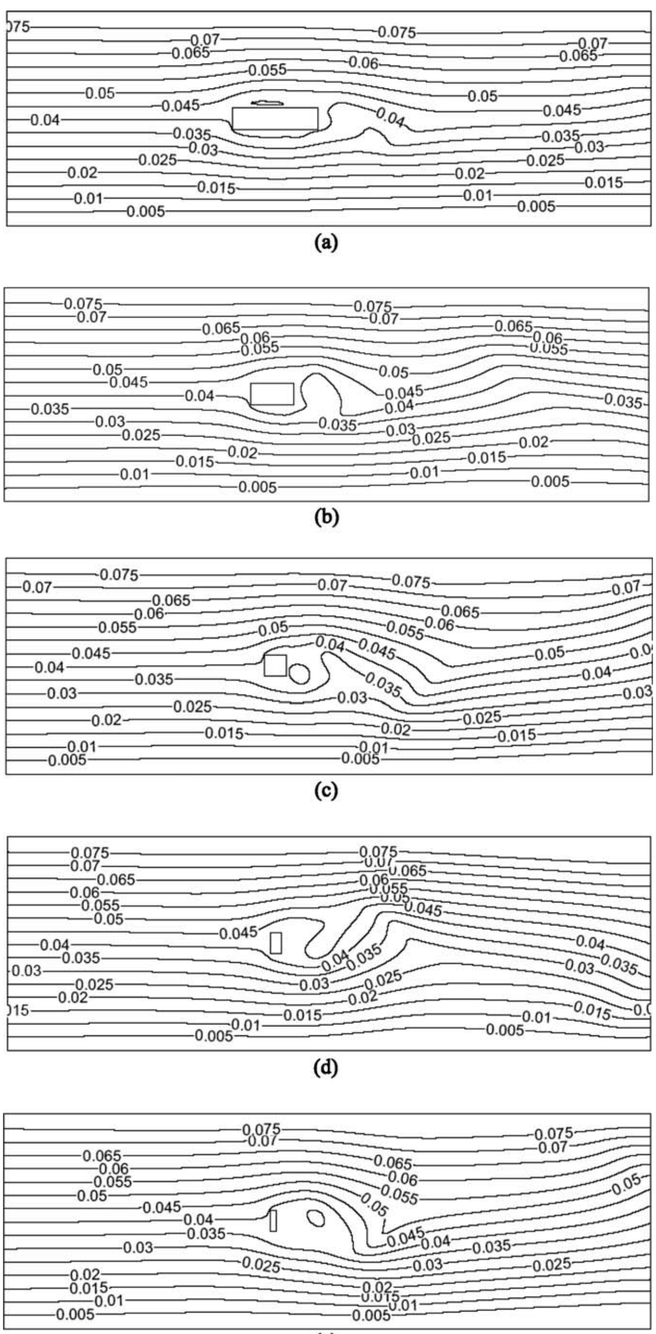

Fig. 2. Streamlines around the object (a) AR = 4, (b) AR = 2, (c) AR = 1, (d) AR = 1/2, (e) AR = 1/4.

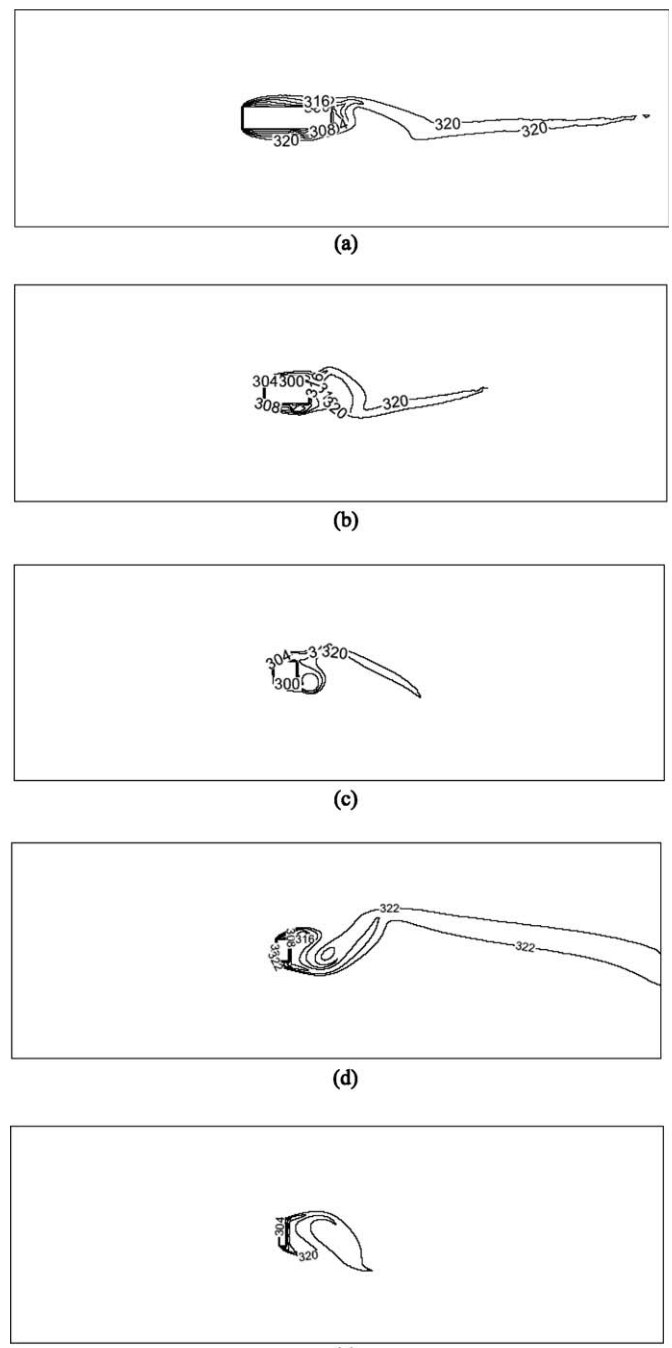

Fig. 3. Static temperature contours around the object (a) AR = 4, (b) AR = 2, (c) AR = 1, (d) AR = 1/2, (e) AR = 1/4.

[16] to produce a grid-independent solution. The boundary conditions assume no-slip conditions for velocity, constant temperature on the surface of the material being dried.

From the temperature fields obtained, the local convection heat transfer coefficient can be determined using

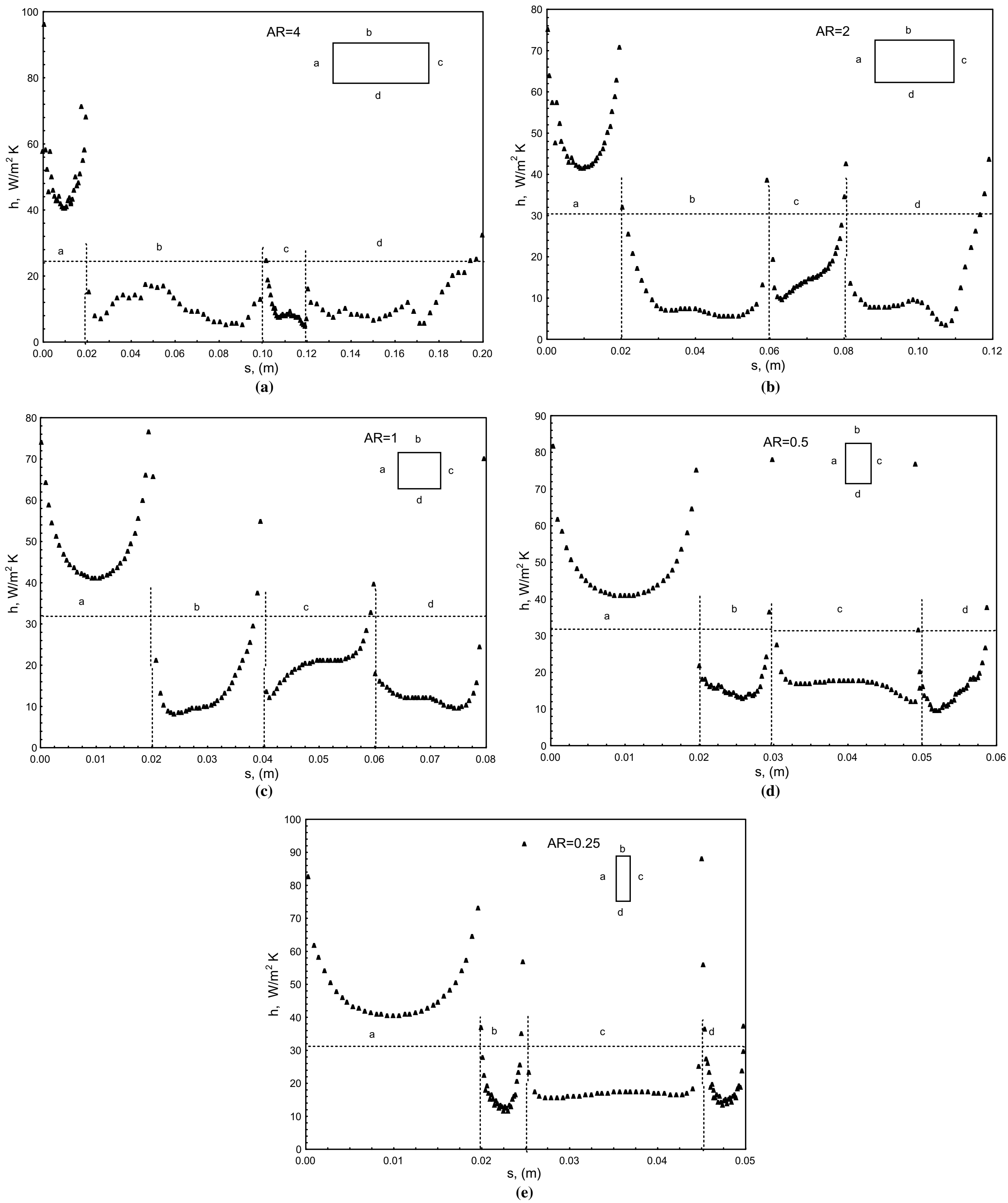

Fig. 4. The variation of h along the surface (a) AR = 4, (b) AR = 2, (c) AR = 1, (d) AR = 1/2, (e) AR = 1/4.

$$-k\frac{\partial T}{\partial n}\bigg|_s = h(T_s - T_\infty), \qquad (5)$$

where s is the coordinate along the surface and n is the normal to the surface.

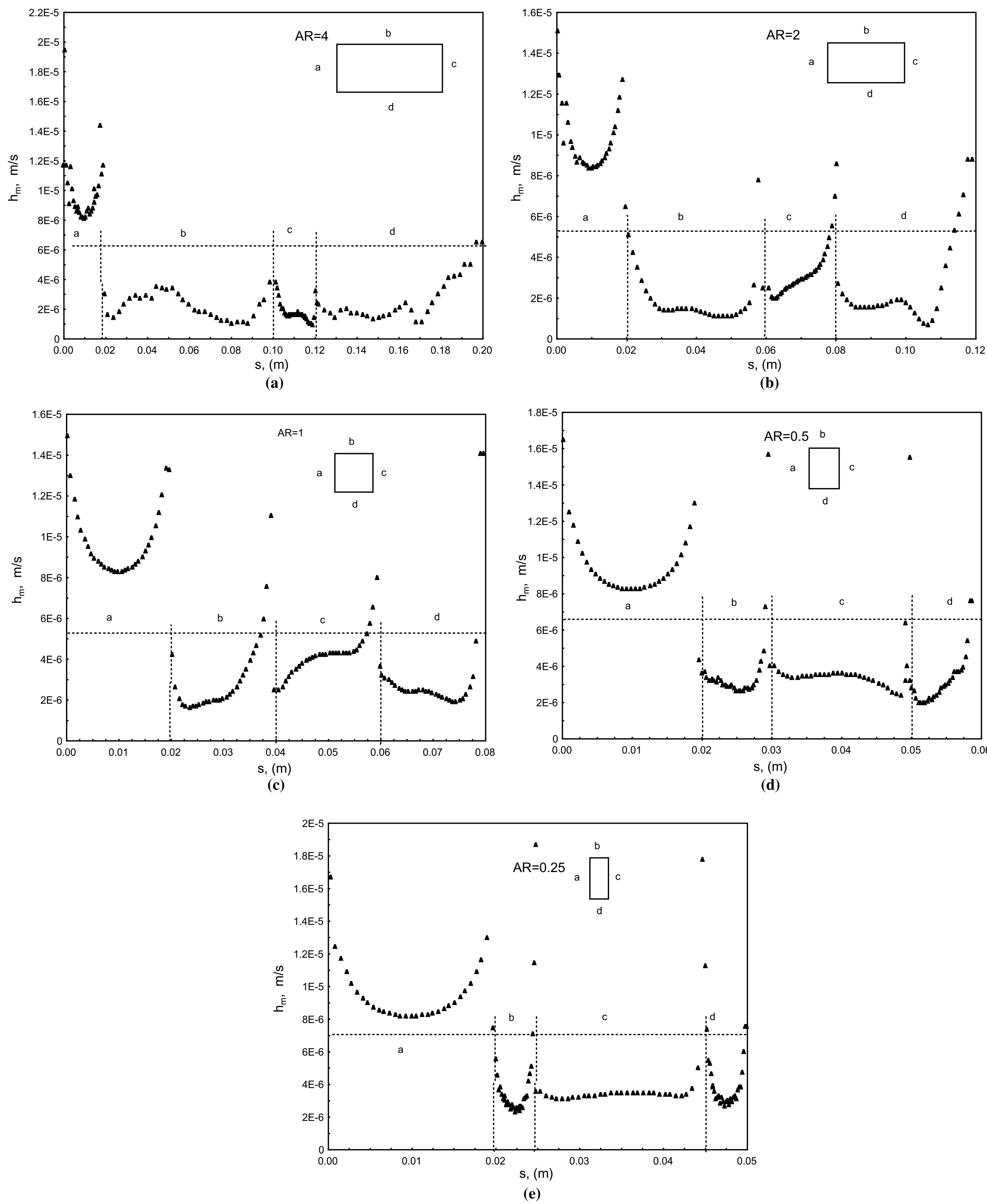

Fig. 5. The variation of h_m along the surface (a) AR = 4, (b) AR = 2, (c) AR = 1, (d) AR = 1/2, (e) AR = 1/4.

After the convective heat transfer, h, is determined using the analogy between the thermal and concentration boundary layers, the convective mass transfer coefficient can be calculated through

$$h_m = h\left(\frac{DLe^n}{k}\right), \tag{6}$$

where Le is the Lewis number representing a measure of the relative thermal and concentration boundary layer thicknesses. For most applications, it is reasonable to assume a value of $n = 1/3$ [18].

2.2. Modeling of internal temperature and moisture fields of the object

In this section, a numerical procedure was developed to analyze heat and mass transfer through diffusion inside the object being dried with some assumptions: (i) moisture content dependent thermophysical properties of the material, (ii) negligible shrinkage or deformation of the material during drying, (iii) negligible heat generation inside the moist object, and (iv) negligible radiation effects.

Under the above assumptions, the following governing 2-D heat and moisture transfer equations can be written:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}, \tag{7}$$

$$\frac{1}{D}\frac{\partial M}{\partial t} = \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \tag{8}$$

with the following initial and boundary conditions for both cases:

$$T(x,y,0) = T_s \quad \text{and} \quad M(x,y,0) = M_s;$$

$$-k\frac{\partial T(0,y,t)}{\partial x} = h(T - T_{air}), \quad -D\frac{\partial M(0,y,t)}{\partial x} = h_m(M - M_{air})$$

at $x = 0$ and $0 \leqslant y \leqslant b$;

$$-k\frac{\partial T(a,y,t)}{\partial x} = h(T - T_{air}), \quad -D\frac{\partial M(a,y,t)}{\partial x} = h_m(M - M_{air})$$

at $x = a$ and $0 \leqslant y \leqslant b$;

$$-k\frac{\partial T(x,0,t)}{\partial y} = h(T - T_{air}), \quad -D\frac{\partial M(x,0,t)}{\partial y} = h_m(M - M_{air})$$

at $y = 0$ and $0 \leqslant y \leqslant a$;

$$-k\frac{\partial T(x,b,t)}{\partial y} = h(T - T_{air}), \quad -D\frac{\partial M(x,b,t)}{\partial y} = h_m(M - M_{air})$$

at $y = b$ and $0 \leqslant x \leqslant a$;

where D is the moisture diffusivity and is obtained from the Arrhenius equation [17] as

$$D = D_0 \exp\left(\frac{-1119}{T}\right). \tag{9}$$

As explained previously, it is assumed that the convective heat and mass transfer coefficients, h and h_m vary along the surface of the material. The distributions of these coefficients have been shown to be predicted from the Fluent simulation. The heat and mass transfer equations given by Eqs. (7) and (8) under the related initial and boundary conditions are solved using the finite difference method. In the solution, the alternating direction method (ADI) [13] is used. In order to prevent the divergence, an under-relaxation parameter is introduced into the heat and mass transfer equations, and its optimum value for both equations is shown to be 0.5. Starting from the initial condition and choosing a suitable time interval, this is continued until steady-state solution exists. For an aspect ratio, AR of 1 (i.e., a square object), the object is discretized by 31×31 of mesh size. The results obtained with this size of the mesh are observed to deviate negligibly from those obtained by a 61×61 mesh size. Therefore, a 31×31 is used for a square object, AR = 1. For the other values of the aspect ratio, the mesh size is refined until the optimum one is reached.

3. Results and discussion

In this part, some simulations for heat and mass transfer are carried out inside a rectangular moist product. Air is considered to be the drying fluid since it is the most common in practice. Five different values of the aspect ratio are considered, AR = 1/4, 1/2, 1, 2, and 4. The height of the object is maintained constant at $H = 0.02$ m, and the length of the object is changed according to the corresponding aspect ratio. Initially, the external flow and temperature fields are analyzed around the object under the process of drying. The moist object is assumed to have constant wall temperature, $T_s = 298$ K and constant wall moisture content, $M_s = 7.196$ kg/kg (db). Here, the drying of the moist slab cut pieces apple with the following thermophysical properties $k = 0.576$ W/m K, $\rho = 856$ kg/m^3, and $C_p = 1929$ J/kg K [17] and the experimental moisture content data of the product were taken from Velic et al. [13].

Note that the moisture diffusivity is considered temperature dependent as given in Eq. (9). In the analysis, these variable moisture diffusivity values are incorporated into the moisture transfer model to study how the moisture transfer coefficient changes with the surface coordinate at different aspect ratios. At the inlet, the following values are considered for the drying air: $T_\infty = 323$ K; $U_\infty = 0.33$ m/s. As an example case, Figs. 2 and 3 illustrate the streamlines and static temperature contours around the object, respectively. As seen from Fig. 2, a wake or vortex shedding region behind the object is observed when the

Table 1
Minimum and maximum values for h and h_m

AR	h		h_m	
	Min	Max	Min	Max
1/4	13.49	92.44	9.63e−06	1.01e−05
1/2	9.50	77.68	9.65e−06	1.01e−05
1	8.05	81.42	9.83e−06	1.49e−05
2	4.33	70.44	9.49e−07	1.27e−05
4	4.60	96.16	9.28e−07	1.94e−05

drying fluid passes through it. The temperature gradients along the surface of the object are not uniform which will result in nonuniform convective heat transfer coefficient distributions along it (Fig. 3). Using Eq. (5), the variation of the convective heat transfer coefficient, h along the surface of the object is obtained for each aspect ratio and shown in Fig. 4. Then, using this knowledge, the variation of the convective mass transfer coefficient, h_m along the surface of the object can be easily determined from Eq. (6), which is illustrated in Fig. 5. As seen from Figs. 4 and 5, for the convection heat and mass transfer coefficients, higher values are obtained at the left-side-wall due

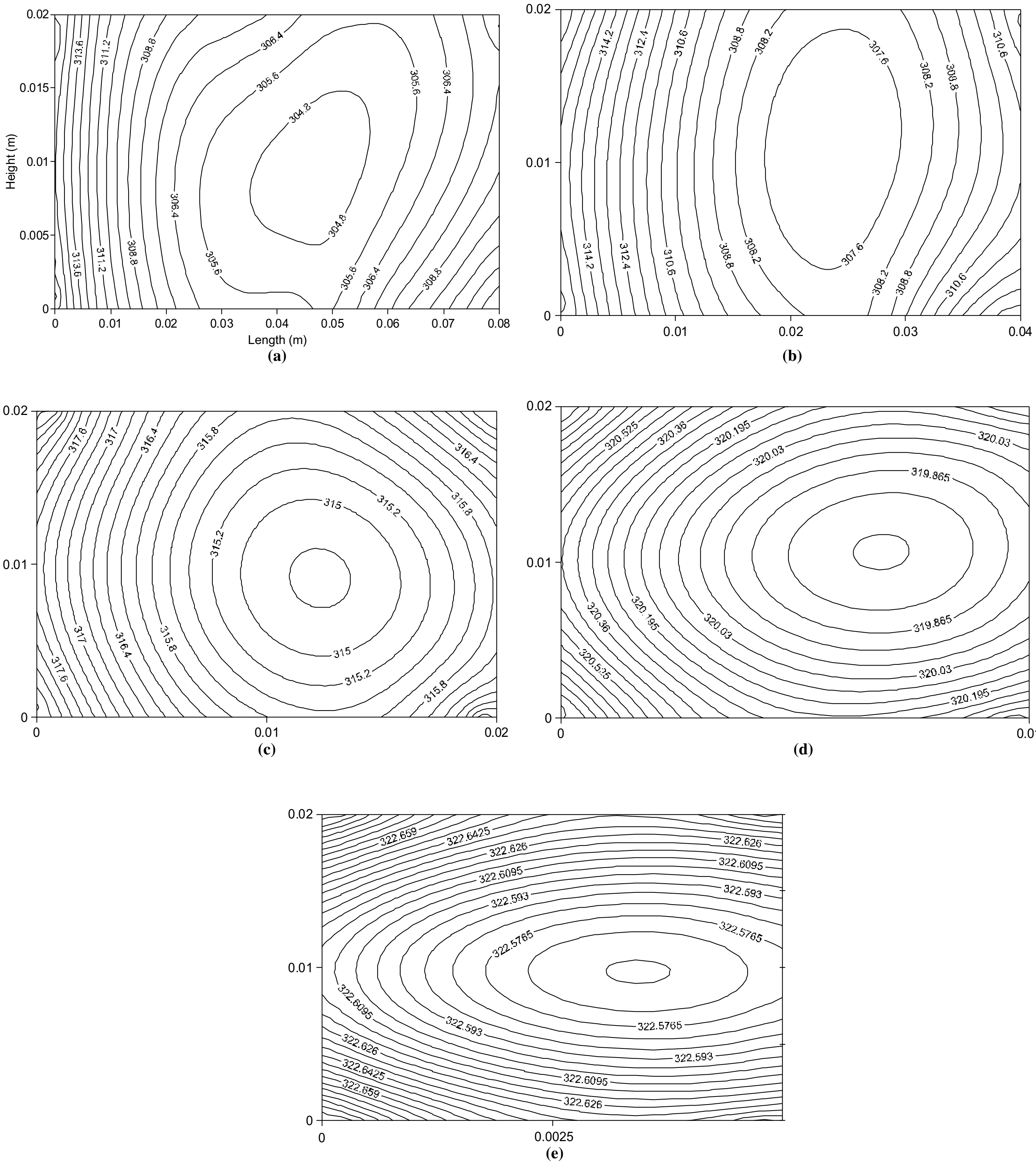

Fig. 6. Temperature distribution inside the object at 500 s (a) AR = 4, (b) AR = 2, (c) AR = 1, (d) AR = 1/2, (e) AR = 1/4.

to the upstream of the drying air while lower values are observed at the right-side-wall due to the vortex shedding. Table 1 presents a summary of some significant results in terms of minimum and maximum heat and mass transfer coefficients at different aspect ratios. This is particularly aimed to provide better presentation and better understanding of the drying phenomena.

Then, the temperature and moisture fields inside the product are analyzed at different times using the code which was developed by the authors. As explained earlier, at this point, the local distributions of the convective heat and mass transfer coefficients are used which were obtained previously in the boundary conditions. The temperature inside the object increases with an increase in the drying

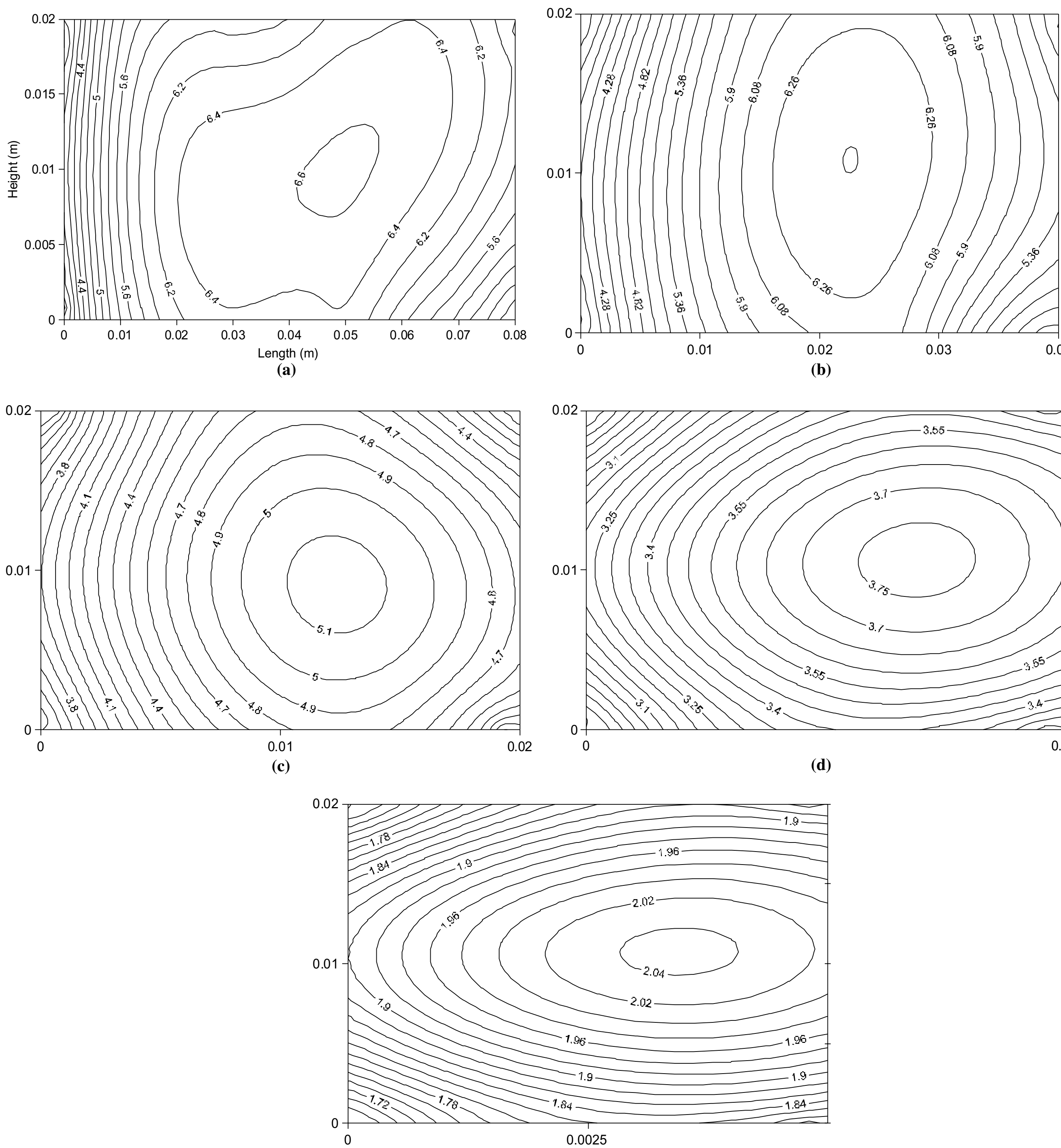

Fig. 7. Moisture distribution inside the object at 500 s (a) AR = 4, (b) AR = 2, (c) AR = 1, (d) AR = 1/2, (e) AR = 1/4.

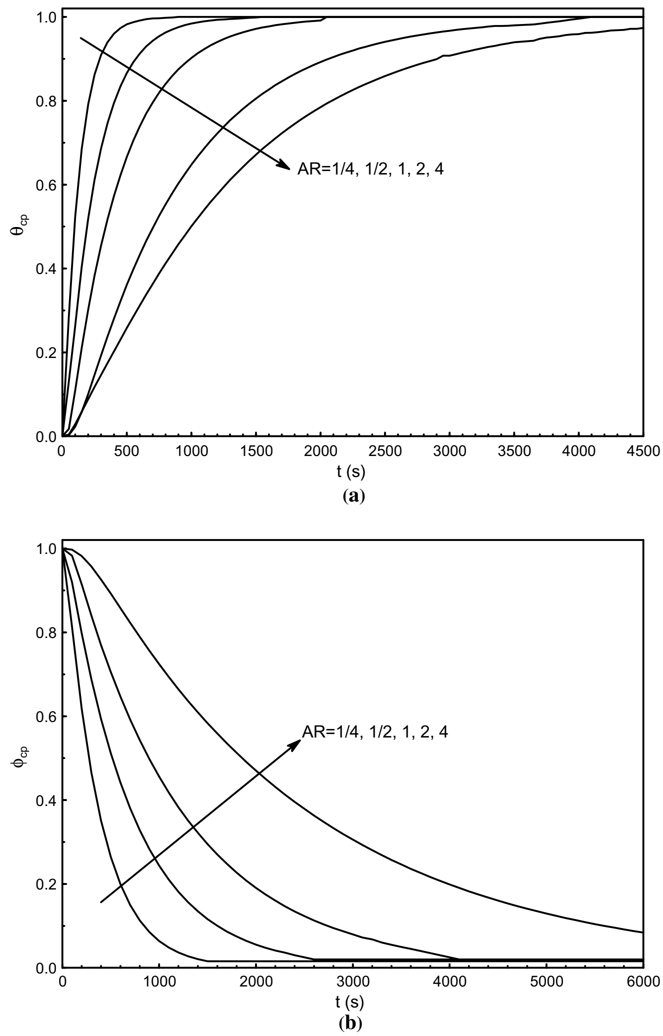

Fig. 8. Effect of the aspect ratio on dimensionless temperature (a) and moisture (b) distributions.

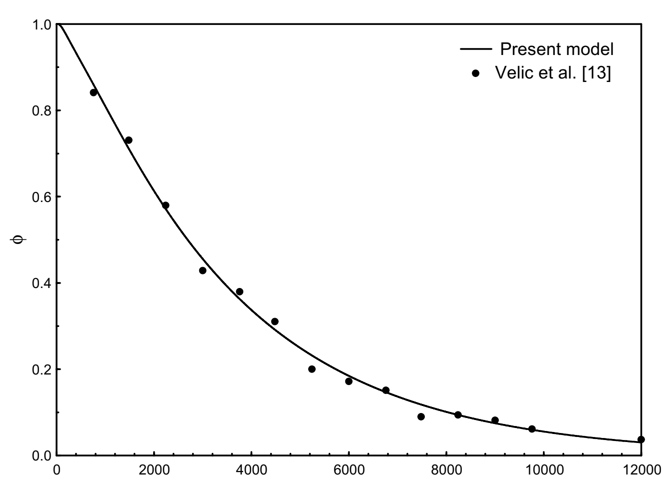

Fig. 9. The variation of the moisture content by drying time.

time since the temperature of the drying air is higher than that of the object. As a result of these transient and non-uniform temperature distributions, the moisture diffusivity which depends on the temperature will vary and in turn the rate of the moisture diffusion inside the object. Figs. 6 and 7 show the temperature and moisture distributions inside the object at 500 s, respectively. As seen the distributions are not symmetrical compared to those observed for the constant convective heat and mass transfer coefficients (see Ref. [14]). Higher temperature and moisture gradients are obtained at the left-side-wall due to the upstream of the drying air. In order to have a better view of the time-variation of temperature and moisture inside the object, variation of the dimensionless temperature and moisture by time at the center point is shown in Fig. 8.

Finally, the code is run, which was developed with the experimental values given by Velic et al. [13], who investigated airflow velocity influence on the kinetics of convection drying of *Jonagold* apple, heat transfer and average effective diffusion coefficients. Fig. 9 shows the variation of the moisture content by drying time for an air velocity of $U_\infty = 0.64$ m/s. As seen, the computation results correspond very well with the experimental ones given by Velic et al. [13].

4. Concluding remarks

In this study, a numerical code was developed for the solution of heat and moisture transfer inside a rectangular moist object under the drying process and incorporated into the CFD package, Fluent. The external flow and temperature fields around the product were first analyzed and later the heat and moisture transfer inside the object. From the temperature gradients obtained through the Fluent analysis, the variations of the convective heat transfer coefficients along the surface of the object were determined. Then, using the analogy between the thermal and concentration boundary layers, the variations of the convective mass transfer coefficients along the surface of the object were predicted. The convective heat and mass transfer coefficients at the left-side-wall have been predicted to be higher than those at the remaining walls due to the upstream of the drying air. It has been also found that the smaller aspect ratios provide shorter drying times than those with higher aspect ratios.

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